Textures in images classification using a multifractal approach

Tahiri Alaoui M[1]., Farssi S.M[2]., Touzani A.[3, 1]
[(1)]UFR : ACSYS, Faculté des sciences –université MOHAMMED V - Rabat, Maroc.
[(2)]Laboratoire de Vision –E.S.P- BP 5085 Dakar, Sénégal.
[(3)]Laboratoire d'Automatique et Informatique Industrielle, E.M.I. - B.P. 765, Agdal, Rabat, Maroc

Abstract

Since ten years, the multifractal analysis represents an original tool in signal and especially image processing. The multifractal methods in texture analysis are becoming efficient due to multifractal modelling and powerful methods in estimation of irregularities functions (exponent of Hölder) and multifractal spectrum. In this respect, we propose a multifractal approach for textural characterization which is based on smoothing Hausdorff multifractal spectrum and by pre-processing that makes the analysis possible and enables us to keep the most important information. This method is a way to extend the one proposed by Piotr & Al **[1]** for textural analysis that processes in analyzing the form and the position of the Legendre multifractal spectrum. Therefore, our method has the advantage to use the Hausdorff multifractal spectrum which provides the best estimation of the fractal dimension related to the differents *fractalities* recovering the image.

Key word: multifractal analysis, smoothing, Hausdorff spectrum, texture analysis.

1. Introduction

With the development of the multifractal formalism **[2][3][4]**, the multifractal approaches for texture analysis have been developed. Generally several methods in texture analysis were issued. Those methods based on multifractal approach exist in two types: the first one uses local information, given by singularity, by building image with singularities and using it to extract statistics describing the presence of differents irregularities of image as like texture attributs. In fact, this idea has been introduced in the segmentation of textured images. Uma and Al **[5]** calculated of statistics on histogram of singularities to characterize SAR* images.
The second kind of methods uses global information given by multifractal spectrum $\left(f(\alpha),\alpha\right)$, where fractal dimension $f(\alpha)$ of differents fractal components E_α have defined each one by a value of singularity α **[6]**. According works of Chaudhuri and Sarker **[7]**, all the multifractal spectrum is considered to classify real images of texture. Piotr and all **[1]** extend this approach for texture attributs by proposing to analyze form and position of Legendre multifractal spectrum of texture. A similar approach using a regional multifractal spetcrum is used to analyze ERS-1** image **[8]**.

* Shuttle Aperture Radar.

** European Radar Satellite.

2. Basic principle of multifractal analysis

The fractal geometry has been introduced to describe the scale relations between geometrical structures and scale for analysis of those structures: the size of a fractal set changes as a scale from which it is examined, given by the fractal dimension **[9]**.
The generalization of the notion of multifractal **[2]** is to consider all sets of multifractal E_α as a hierarchy of sets in which, each of them has its own fractal dimension $f(\alpha)$. Thus multifractal analysis gives a scale relation that requires a family of dimensions instead of one dimension as in fractal geometry case **[10][11]**.
Multifractal in images analysis consists on defining measures from gray level to calculate spectrums and to process each point based on local α and global informations $f(\alpha)$.

3. Texture analysis

3.1 The smoothing of the Hausdorff multifractal spectrum

Smoothing of discretes curves consists on a substitution of those by regular form from which one has good representation to access for analyzing them. The filters 1-D as median, average stage and its derivates are most used for smoothing functions 1-D like signal in order to eleminate noise. But size and the number of iterations that one has to use those filters represent parameters to regulate for good results.

Pratically, a criteria of quality based on a distance of the curve to points (or in total quadratic error) is used to determine a regular function that realise an optimale smoothing.

Considering n points (t_i, y_i) $t_i \in [a,b]$, a function thus should be found $f(t)$ to minimize **Eq1** or **Eq2** defined as follow:

$$E_1 = \sum_{i=1}^{n} \left(f(t_i) - y_i \right)^2 \qquad \textbf{(Eq1)}$$

$$ou \quad E_2 = \underset{i}{Max} \left\{ \left| f(t_i) - y_i \right| \right\} \qquad \textbf{(Eq2)}$$

Criteria of (Eq1) and (Eq2) permit to valorize results given by those two representatives' classes of filters that exist: linear filters (average) and non linear filters (median).

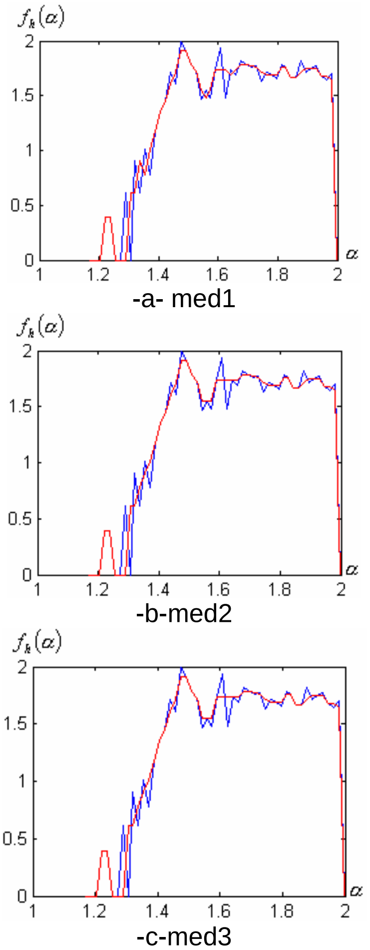

Figure 1: The smoothing of multifractal spectrum with the median filter 1D (size 1x3). a- The median filter applied once. b- Twice. c- Three times.

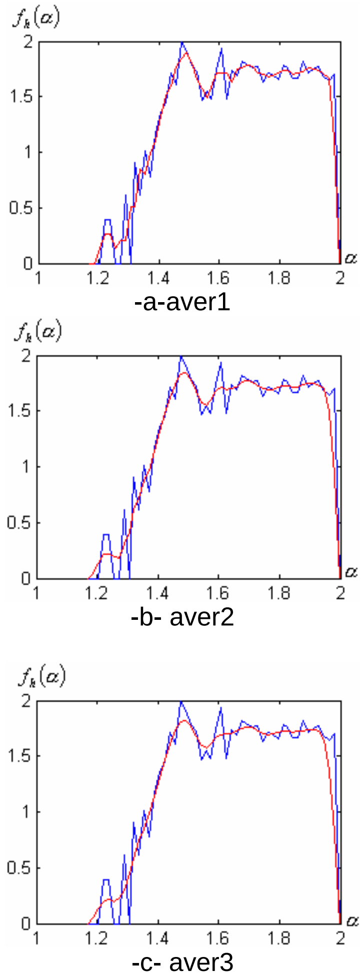

Figure 2: The smoothing of multifractal spectrum with the average filter1D(size 1x3). a- The average filter apply once. b- Twice. c- Three times.

We notice that the curve obtained from an average stage is smooth but the curve obtained from a median filter presents some angulars points. In facts, we consider criterea (Eq1) because it takes in consideration all points of the curve. Find results are given in the table (**table 1**) which gives the average, standard deviation and the coefficient of variation of quantity E_1 measured from spectrum of a set of 40 images chosen.

The analysis of results prove that the average stage filter offer a regular curve form that one can use for texture of image analysis. Similar results had been obtained while using the criteria (Eq2).

	Median	Average
Average	0.924	0.941
Dev. Std.	0.205	0.221
Coef. Var.	22.18	34.74

Table 1 : Summary of results with respect to a criteria (Eq1) for a median filter and average filter.

The characterization of texture by analysing the form and the position of the smoothed multifractal spectrum is now possible by using Hausdorff multifractal spectrum which represents the most exat spectrum to estimate fractal dimension for various fractalities recovering the image.

3.2 The fractal dimension, average D_a and central D_C

The image analysis using multifractal spectrum is at the same time easy and difficult. In fact, it is easy because the distinction of homogenous regions can be realised by selection of subregions of image that has for fractal dimension a value toward 2 whereas edge points are characterized by fractal dimension neighbourhood of 1. Otherwise it is difficult to explain subregions with fractal dimension different of 1 and 2, that means fractal set E_α that dimension is $f(\alpha) << 1$ or $1 << f(\alpha) << 2$. The characterization of an image by homogenous regions $(f(\alpha) \approx 2)$ has a problem of limit because there is no general rule which gives the value that one has homogenous regions from multifractal spectrum.
That's why fractalities on image have to be divided in two groups:

- All given subsets E_α defined by $f_S < f(\alpha) \leq 2$ (with $1 < \alpha_S$) having homogenous regions. This set is representative because if the texture is smooth its contribution in multifractal spectrum becomes interesting but it is the reverse when the texture is not smooth.
- All given subsets E_α defined $f(\alpha) \leq f_S$ has an irregularity that corresponds to the edges.

In fact, cutting the spectrum in two can be realised using average fractal dimension defined in the same way that one definies average in physics values. In that case it is difficult to give a signification apart of representing a statistic sampling describes value $f_h(\alpha)$. In our case we show how this statistic can be important in classification of images of texture.

$$D_m = \frac{1}{M} \sum_{\alpha=\alpha_{\min}}^{\alpha_{\max}} f_h(\alpha)$$

We have two subregions cover the image:

- The pixels (i,j) with Holder exponent $\alpha(i,j)$ is that : $D_m < f_h(\alpha(i,j)) \leq 2$.
- The pixels (i,j) with Holder exponent $\alpha(i,j)$ is that : $f_h(\alpha(i,j)) \leq D_m$.

All spectrum points given by $D_m < f_h(\alpha(i,j)) \leq 2$ belongs to an intervalle of singularities by those values:

$$\alpha^m_{\min} = \min\{\alpha / f_h(\alpha) = D_m\}$$
$$\alpha^m_{\max} = \max\{\alpha / f_h(\alpha) = D_m\}$$

The central holder exponent α_C is defined by average of $\alpha \in [\alpha^m_{\min}, \alpha^m_{\max}]$. Consequently, the D_C has to be defined with respect to α_C where:

$$\alpha_C = \frac{1}{N} \sum_{\alpha=\alpha^m_{\min}}^{\alpha^m_{\max}} \alpha$$

Figure 3 gives differents quantities of multifractal spectrum.

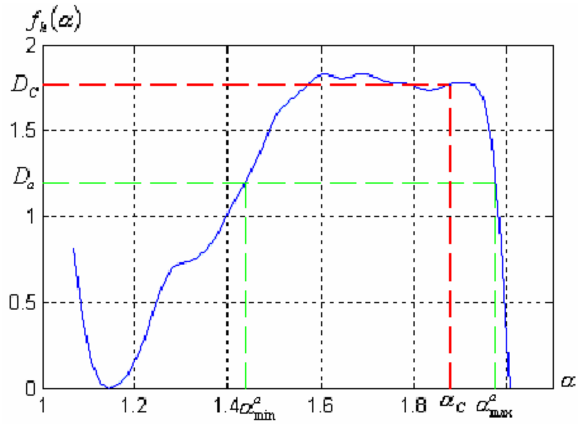

Figure 3 : Exemple of smoothed multifractal spectrum.

4 Attributs of texture

The spectrum being divided in two zones : left $\alpha < \alpha_C$ and right $\alpha_C \leq \alpha$, then one can prove that the spectrum being with respect to those two types of irregularities can be element which characterizes the texture.
The position of spectrum is defined by singularities maximum $\alpha_{\max}$, minimum $\alpha_{\min}$ and central singularity α_C, but the form can be quantified by the calcul of area of caracteristics of differents parts of spectrum and dispatching with respect to two axis α and $f_h(\alpha)$.
So attributs of texture are resumed hereafter.

4.1 Analysis of total spectrum

- Total area :

$$att1_h = \int_{\alpha_{min}}^{\alpha_C} f_h(\alpha)d\alpha \qquad att2_h = \int_{\alpha_C}^{\alpha_{max}} f_h(\alpha)d\alpha$$

$$att3_h = {att1_h}/{att2_h}$$

$att1_h$ (Respectively $att2_h$) gives the information on fractalities, defined by Holder exponents $\alpha < \alpha_C$ (respectively $\alpha_C \le \alpha$) throw the distribution of their dimensions. While $att3_h$ gives the information on types of fractalities with respect to the other fractal dimension.

- Average integral on α axis and their proportion:

$$att4_h = att1_h / (\alpha_C - \alpha_{\min}) \qquad att5_h = att2_h / (\alpha_{\max} - \alpha_C)$$

$$att6_h = att4_h / att5_h$$

$att4_h$ **(resp.** $att5_h$**)** gives informations on the value of the dimension, which is constante on the intervalle $[\alpha_{\min}, \alpha_C[$ (resp. $[\alpha_C, \alpha_{\max}]$), occupies the same area $att1_h$ (respectively $att2_h$) of $f_h(\alpha)$ on the same intervalle. That value depends on spectrum being to the support $[\alpha_{\min}, \alpha_C[$ (resp. $[\alpha_C, \alpha_{\max}]$). While $att6_h$ gives the information on the importance of quantities $att4_h$ and $att5_h$ one with respect to the other.

- Percentage of total area :

given $f^l_{\min} = \min\{f_h(\alpha) / \alpha \in [\alpha_{\min}, \alpha_C[\}$ and $f^r_{\min} = \min\{f_h(\alpha) / \alpha \in [\alpha_C, \alpha_{\max}]\}$

$f^l_{\max} = \max\{f_h(\alpha) / \alpha \in [\alpha_{\min}, \alpha_C[\}$ and $f^r_{\max} = \max\{f_h(\alpha) / \alpha \in [\alpha_C, \alpha_{\max}]\}$

l : left and r : right

then we can definie those quantities:

$$att7_h = att4_h / (f^g_{\max} - f^g_{\min}) \qquad att8_h = att5_h / (f^r_{\max} - f^r_{\min})$$

$$att9_h = att7_h / att8_h$$

$att7_h$ represent the percentage of the total area in the area of the too small rectangle contening the left part of the spectrum. In fact, this quantity gives informations on the position and the form of the spectrum on its left part because as left part of the spectrum is highest, as the attribut $att7_h$ shall be important. The very kind interpretation can be done on the right part of the spectrum. While $att9_h$ gives informations on a ratio $att7_h$ and $att8_h$.

4.2 Partial analysis of spectrum

The spectrum has an important part where the fractal dimension has big values. We have estalished the method allowing to estimate this area using the average of the dimension.

Now we are going to analyse the given part of the spectrum using the Holder exponent to the intervalle $[\alpha^m_{\min}, \alpha^m_{\max}]$ inside homogenous regions of the texture. First of all, we define similars attributs to see how will be the spectrum inside the two parts of those intervalles $[\alpha^m_{\min}, \alpha_C]$ and $[\alpha_C, \alpha^m_{\max}]$.

So that partials area and derived quantities have been established.

- Partials area :

Given:

$f^{l,m}_{\min} = \min\{f_h(\alpha) / \alpha \in [\alpha^m_{\min}, \alpha_C]\}$ and

$f^{r,m}_{\min} = \min\{f_h(\alpha) / \alpha \in [\alpha_C, \alpha^m_{\max}]\}$

l: left and r : right

then we define those areas:

$$att10_h = \int_{\alpha^m_{\min}}^{\alpha_C} f_h(\alpha)d\alpha - (\alpha_C - \alpha^m_{\min}) f^{l,m}_{\min}$$

$$att11_h = \int_{\alpha_C}^{\alpha^m_{\max}} f_h(\alpha)d\alpha - (\alpha^m_{\max} - \alpha_C) f^{r,m}_{\min}$$

$$att12_h = att10_h / att11_h$$

$att10_h$ (resp. $att11_h$) gives informations on the form of the left part of that important region of the spectrum because this attribut gives the area of the spectrum in the nearst repair defined by singularities $[\alpha^m_{\min}, \alpha_C]$ (resp. $[\alpha_C, \alpha^m_{\max}]$). While $att12_h$ gives an idea on the importance of the ratio $att10_h$ and $att11_h$.

- Averages integrals on α axis and their proportion :

Given the following values:

$f^{l,m}_{\max} = \max\{f_h(\alpha) / \alpha \in [\alpha^m_{\min}, \alpha_C[\}$ and

$f^{r,m}_{\min} = \min\{f_h(\alpha) / \alpha \in [\alpha_C, \alpha^m_{\max}]\}$

then given the following ratio :

$$att13_h = att10_h / (\alpha_C - \alpha^m_{\min}) \qquad att14_h = att11_h / (\alpha^m_{\max} - \alpha_C)$$

$$att15_h = att13_h / att14_h$$

$att13_h$ (respectively $att14_h$) gives the formations on the value of dimension which is constant on the intervalle $\left[\alpha_{\min}^{m}, \alpha_C\right[$ (resp. $\left[\alpha_C, \alpha_{\max}^{m}\right]$), occupies the same area with $att10_h$ (resp. $att11_h$) of $f_h(\alpha)$ on the intervalle $\left[\alpha_{\min}^{m}, \alpha_C\right[$ (resp. $\left[\alpha_C, \alpha_{\max}^{m}\right]$). This value depends on the spectrum being on its support $\left[\alpha_{\min}, \alpha_C\right[$ inside the repair of the axis $\left[\left(\alpha_{\min}^{m}, f_{\min}^{l,m}\right)\left(\alpha_C, f_{\min}^{l,m}\right)\right)$ and $\left[\left(\alpha_{\min}^{m}, f_{\min}^{l,m}\right)\left(\alpha_{\min}^{m}, f_{\max}^{l,m}\right)\right)$. This value gives informations on homogenous level of the texture having dimension near and less than 2. While $att15_h$ gives informations on homogenous level of $att13_h$ and $att14_h$ one with respect to the other.

- Percentage of partial area :

$$att16_h = att13_h \big/ \left(f_{\max}^{l,m} - f_{\min}^{l,m}\right) \qquad att17_h = att14_h \big/ \left(f_{\max}^{r,m} - f_{\min}^{r,m}\right)$$

$$att18_h = att16_h \big/ att17_h$$

$att16_h$ (resp. $att16_h$) represents the percentage of the partial area $att10_h$ (resp. $att11_h$) in the area of the too small rectangle contening the left part (resp. right) of the central area of the spectrum. This quantity gives informations on the position and the form of the spectrum inside the left part because as the left part of the spectrum is highest, as the attribut $att16_h$ shall be important. While $att18_h$ gives the informations on a ratio $att16_h$ with respect to $att17_h$.

5 Results and discussion

To improve our approach, we propose the classification of some images of natural texture of free database Meastex chosen by the most of authors for analysis of texture and approaches tests. From the database, let us consider a learning set and test one.

All used images can be homogenous and isotrope. Those images consist on 12 classes of 512x512 as **figure 4**. Each image indexed by an attribut of arbitrary class (class1, class2, …) and visual images non differenciable are closed in the same class. From each class, 16 images that have the size 128x128 are extracted in order to have 192 images belonging 12 classes. But each image has attributs calculated as above.

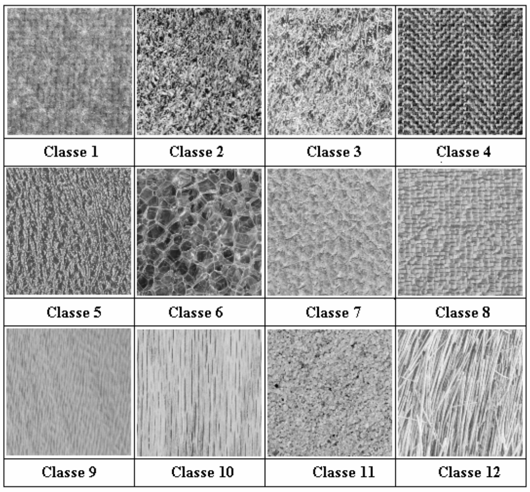

Figure 4 : Images chosen from Meastex base and distributed on 12 classes.

		Waited												
	CLASSE	01	02	03	04	05	06	07	08	09	10	11	12	TOTAL
TESTS	01	7	0	2	0	1	0	0	0	0	0	0	0	10
	02	1	9	0	0	0	0	0	0	0	0	0	0	10
	03	1	1	8	0	0	0	0	0	0	0	0	0	10
	04	0	0	0	10	0	0	0	0	0	0	0	0	10
	05	0	0	0	2	8	0	0	0	0	0	0	0	10
	06	0	0	0	0	0	8	2	0	0	0	0	0	10
	07	0	0	0	0	0	0	9	1	0	0	0	0	10
	08	0	0	0	0	0	0	1	8	0	0	1	0	10
	09	0	0	0	0	0	0	0	0	8	1	0	1	10
	10	0	0	0	0	0	0	0	0	2	7	0	1	10
	11	0	0	0	0	0	0	0	1	1	0	8	0	10
	12	0	0	0	0	0	0	0	0	0	0	0	10	10
Ratio of classification														83.33%

Table 1 : Results of classification of differents images tests.

While distributing all images in two groups - learning and test - we learn to our system the behaviour of each class. The classification is based on k nearest neighbour (k-nn) **[12]**. The classification results from all tests are given in **table 2**. We notice that a good ratio of the classification is about 83.33%.

Bibliography

[1] P. STANCZYK, P. SHARPE, *Classification of Natural Images from Shape Analysis of the Legendre Multifractal Spectrum,* in Dekking et al, Fractals: Theory and Applications in Engineering, pp. 67-79, Springer-Verlag, London, 1999.

[2] J. LEVY VEHEL, R. VOJAK, *Multifractal analysis of Choquet Capacities :preliminary results,* rapport de recherche de INRIA, RR-2576, Juin 1995.

[3] J. P. BERROIR, 1994. *Analyse Multifractale d'Images.* Thèse de doctorat de l'Université Paris IX. Paris, France

[4] J. LEVY VEHEL, P. MIGNOT, and J.-P. BERRIOR. *Texture and multifractals: New tools for image analysis.* Technical Report 1706, Institut National de Recherche en Informatique et en Automatique, 1992.

[5] U. S. RANJAN and A. NARAYANA, "*Classification Of Objects In SAR Images Using Scaling Feature"*", ICVGIP, December 16-18, Space Applications Centre Ahmedaba, 2002.

[6] SAUCIER, A., RICHER, J. et MULLER, J. *Assessing the scope of the multifractal approach to textural characterization with statistical reconstruction of images*. *Physica*, vol.A 311, p.231-259, 2002.

[7] B.B. CHAUDHURI and N. SARKAR, "*Texture segmentation using fractal dimension*", IEEE. Trans. on Pattern Analysis and Machine Intelligence, Vol. 17, No. 1, pp. 72-77, 1995.

[8] P. SAILHAC, F. SEYLER, *Texture characterization of ERS-1 images by regional multifractal analysis*, Springer-Verlag. pp. 32-41, 1997.

[9] B.B. MANDELBROT, *Fractal Geometry of Nature*, San Francisco, CA Freeman 1982.

[10] PENTLAND, A. 1984. *Fractal Based Description of Natural Scenes*. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, Vol. PAMI-6, pp. 661-674.

[11] KAPLAN, L.M. 1999. *Extended Fractal Analysis for Texture Classification and Segmentation*. IEEE Transaction on Image Processing, Vol.8, No. 11, pp. 1572-1585.

[12] T. HASTIE and R. TIBSHIRANI, *Discriminant Adaptive Nearest Neighbor Classification*" IEEE Trans. Pattern Anal. Mach. Intell., Vol. 18, N. 6, pp. 607-616, 1996.